PSC 2020/21 (375AA, 9CFU)

Principles for Software Composition

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Exercises #8

Probabilities

[**Ex. 1**] A gambler wins a considerable amount of money by betting repeatedly that in four rolls of a die at least one six would turn up. To get more people to play, he decides to change the game to bet that in N rolls of two dice a pair of six will show up. What is the least value of N that is favourable to the gambler?

Ex. 1, rolling dice

No six in one roll has probability $\frac{5}{6}$

in four independent rolls $\left(\frac{5}{6}\right)^4 = 0.48$

No pair of six in one roll has probability $\frac{35}{36}$

in N independent rolls $\left(\frac{35}{36}\right)^N$

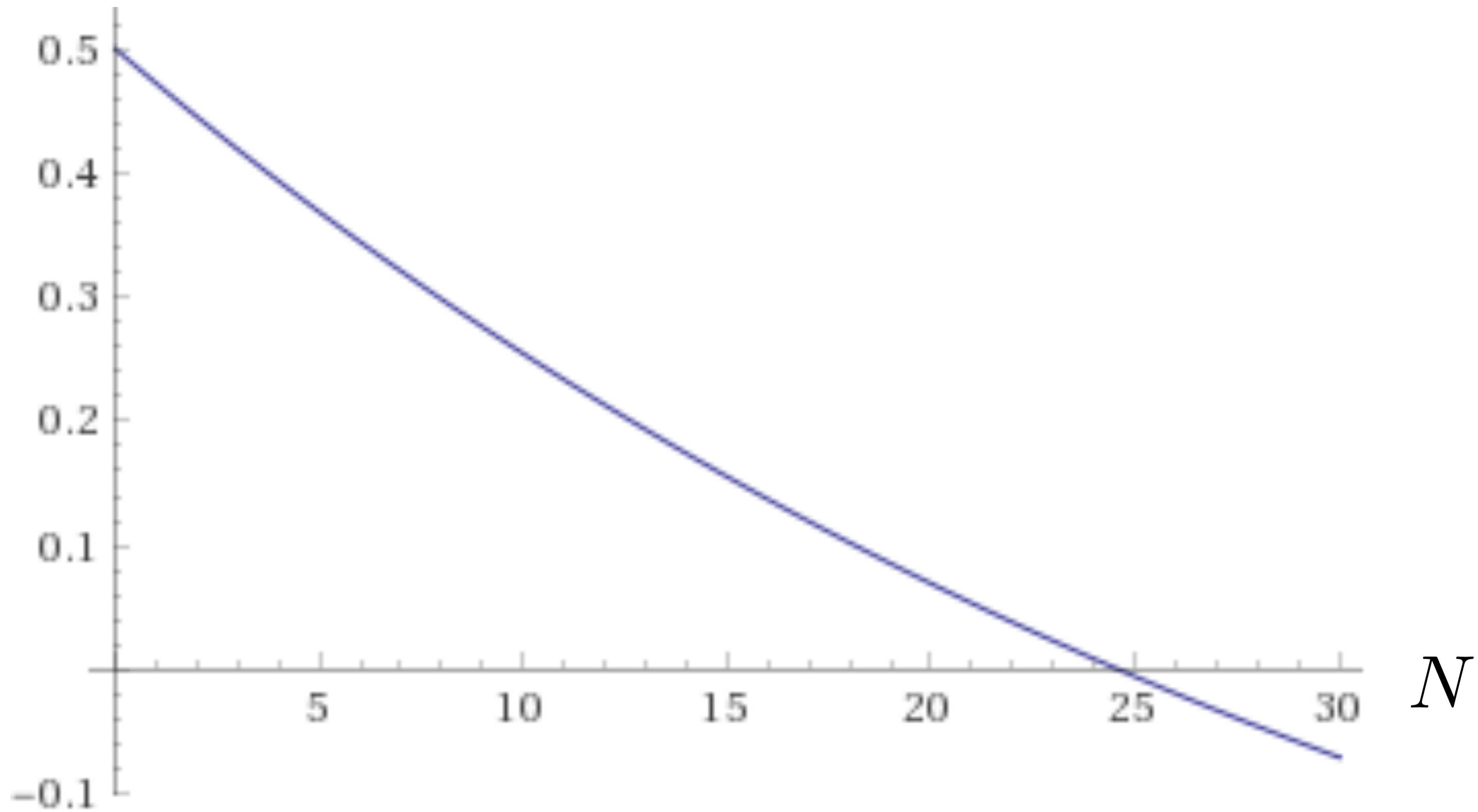
favourable to gambler: $\left(\frac{35}{36}\right)^N < 0.5$

$$N \cdot \log_2\left(\frac{35}{36}\right) < \log_2(0.5) \qquad N \cdot (\log_2(35) - \log_2(36)) < -1$$

$$N > \frac{1}{\log_2(36) - \log_2(35)} = 24.6 \qquad \boxed{N = 25}$$

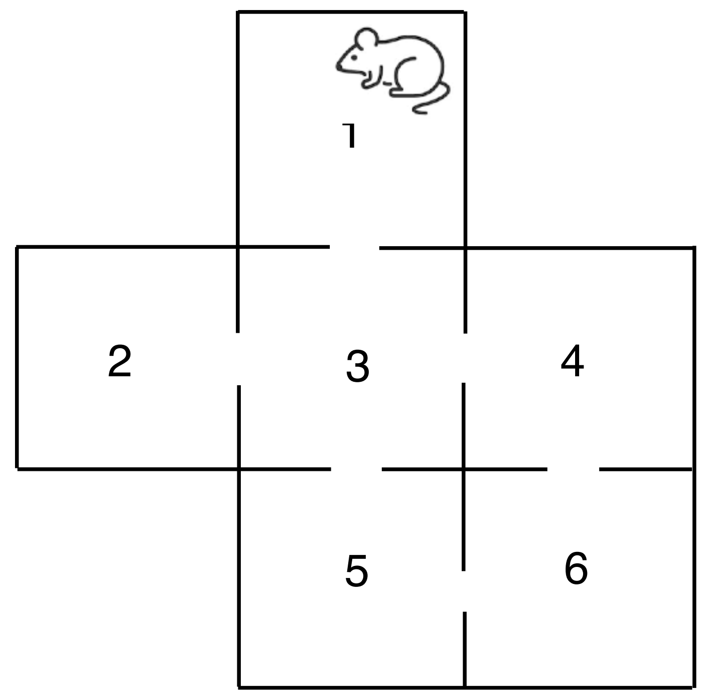
Ex. 1, rolling dice

$$\left(\frac{35}{36}\right)^N - 0.5$$



DTMC

[**Ex. 2**] A mouse runs through the maze shown below. At each step it stays in the room or it leaves the room by choosing at random one of the doors (all choices have equal probability).

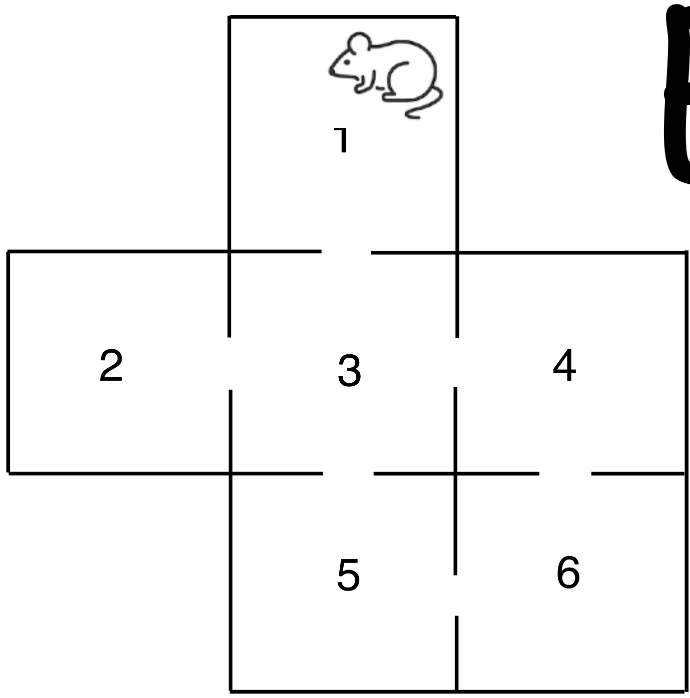


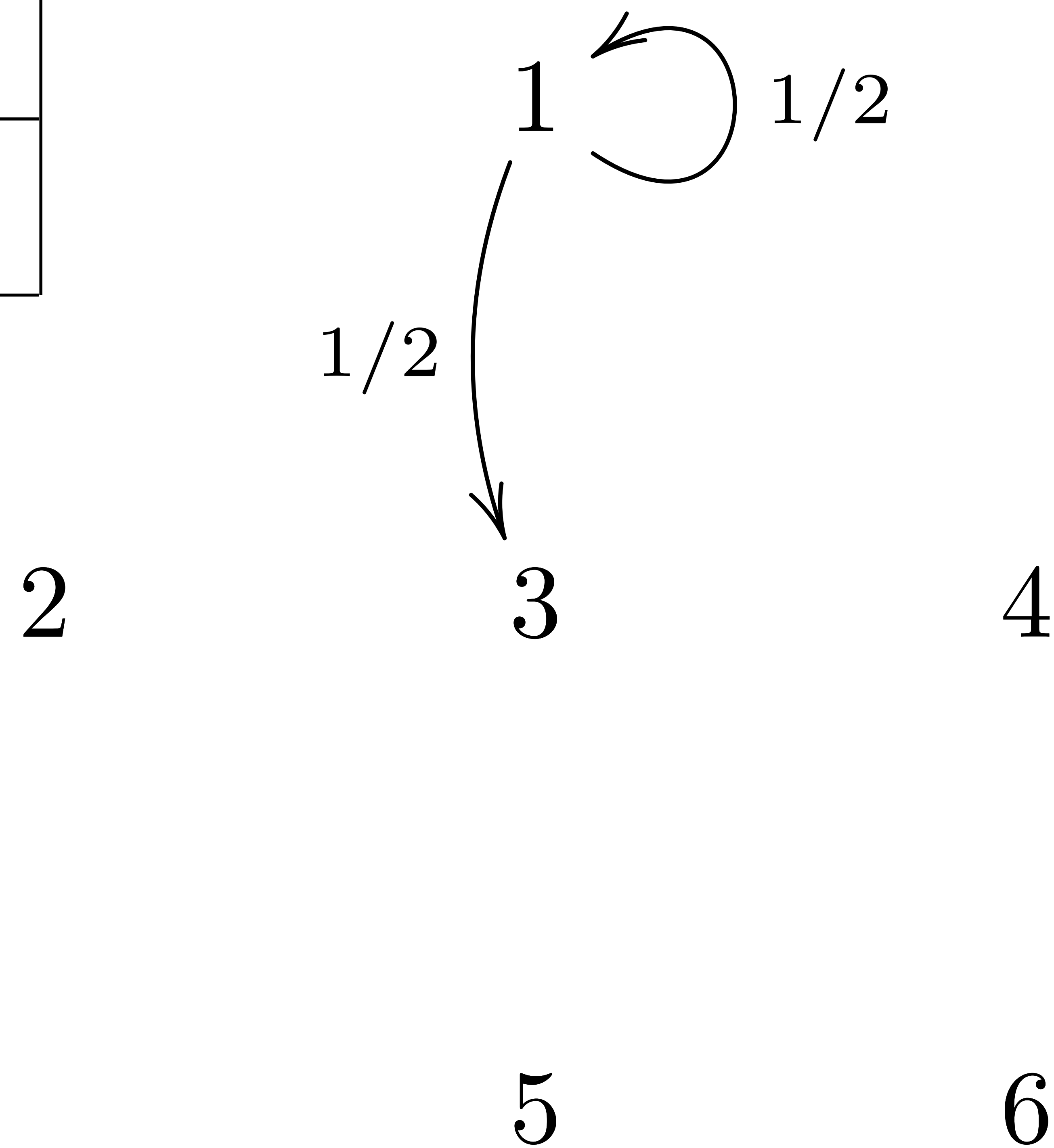
1. Draw the transition graph and give the matrix P for this DTMC.

2. Show that it is ergodic and compute the steady state distribution.

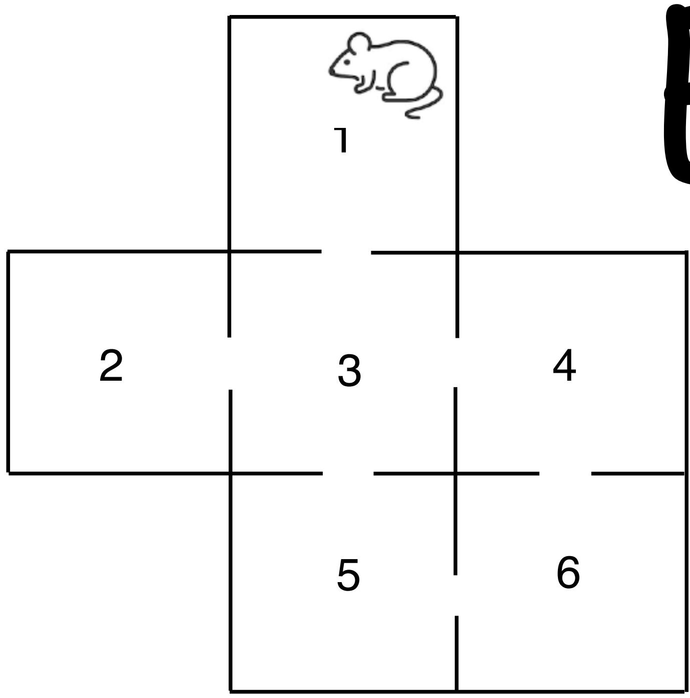
3. Assuming the mouse is initially in room 1, what is the probability that it is in room 6 after three steps?

Ex. 2, mouse



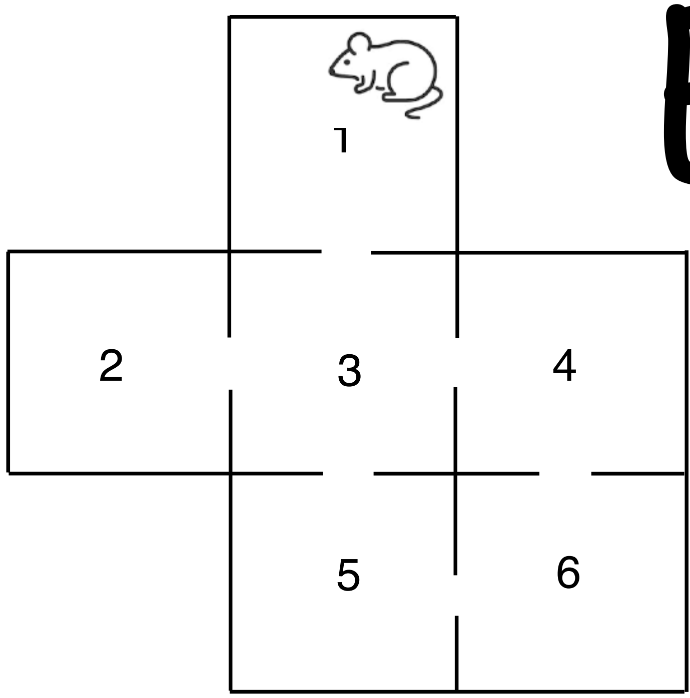


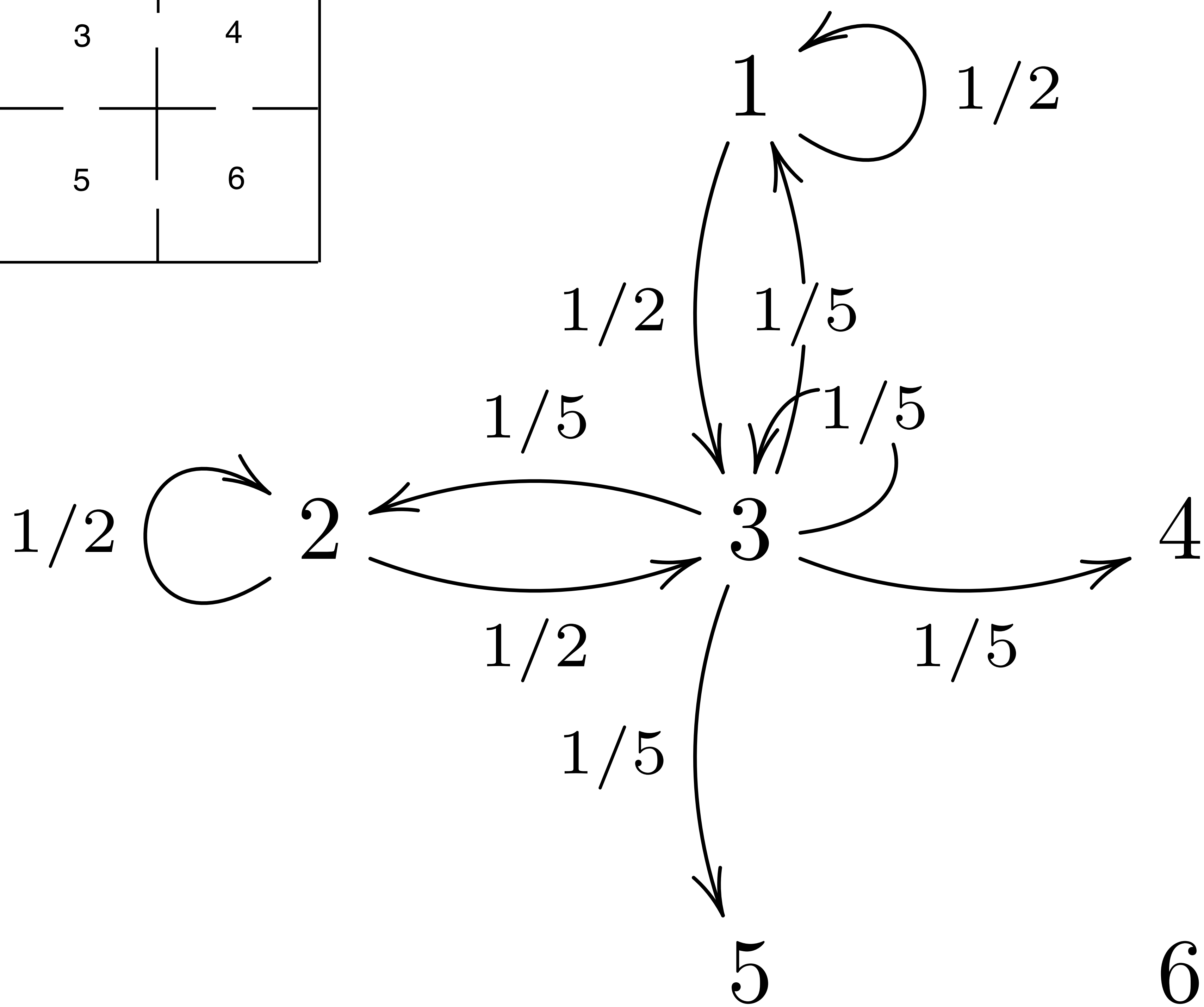
Ex. 2, mouse



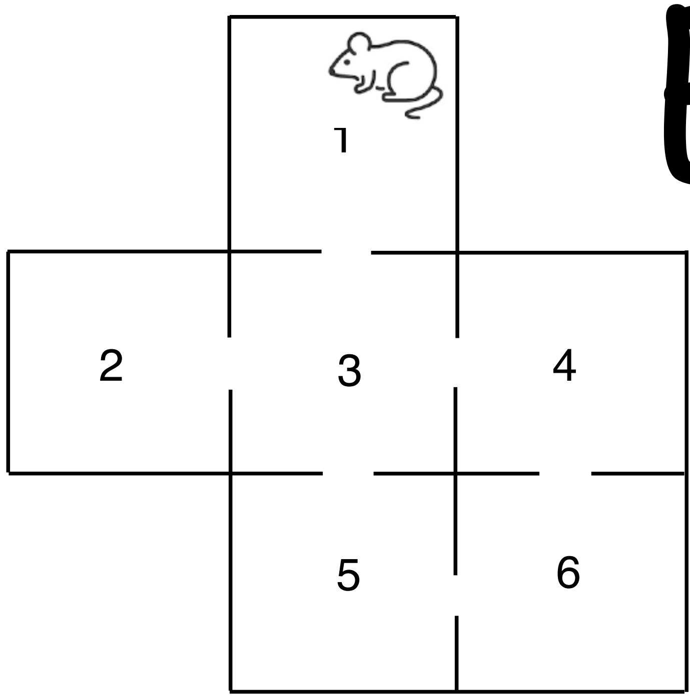
$$
\begin{tikzcd}
 & 1 \arrow[loop right, "1/2"] \arrow[d, "1/2"'] & \\
2 \arrow[loop left, "1/2"] \arrow[r, "1/2"'] & 3 & 4 \\
 & 5 & 6
\end{tikzcd}
$$

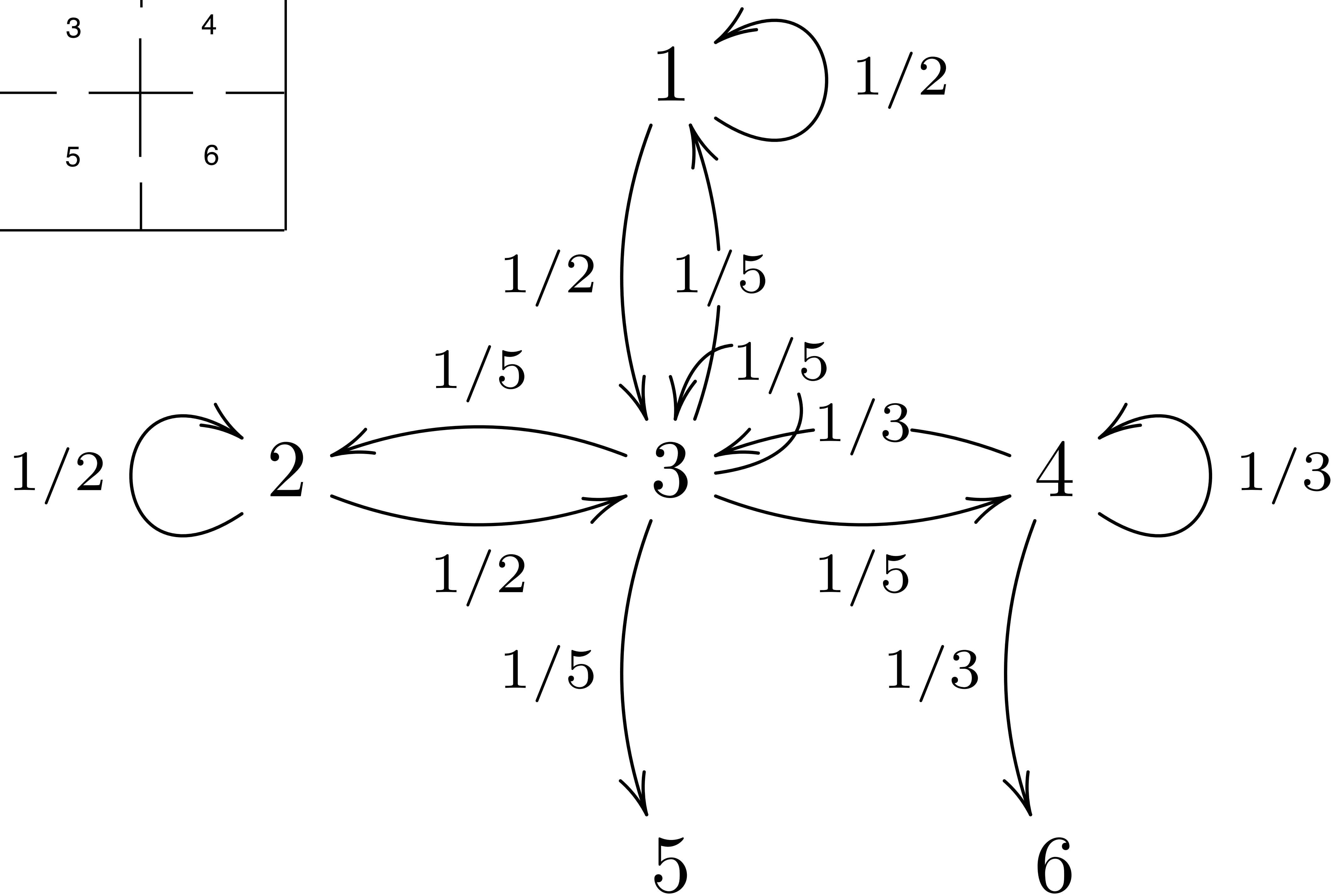
Ex. 2, mouse



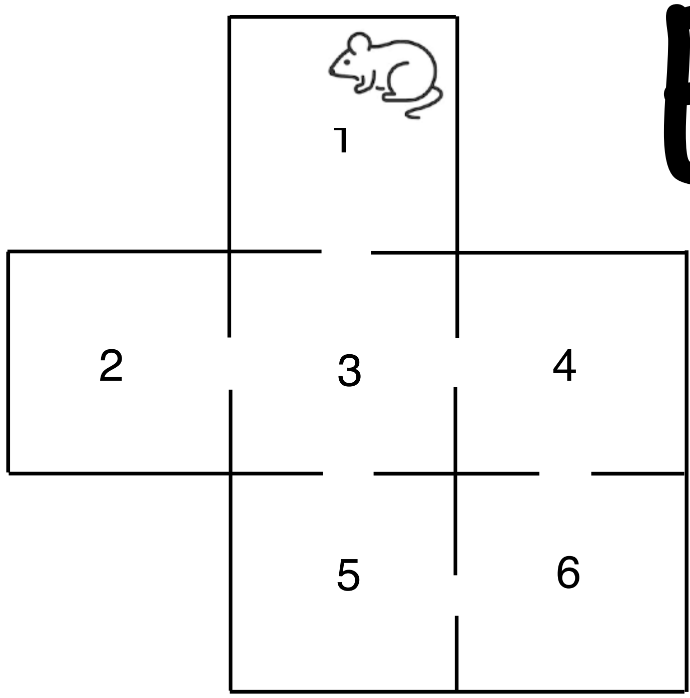


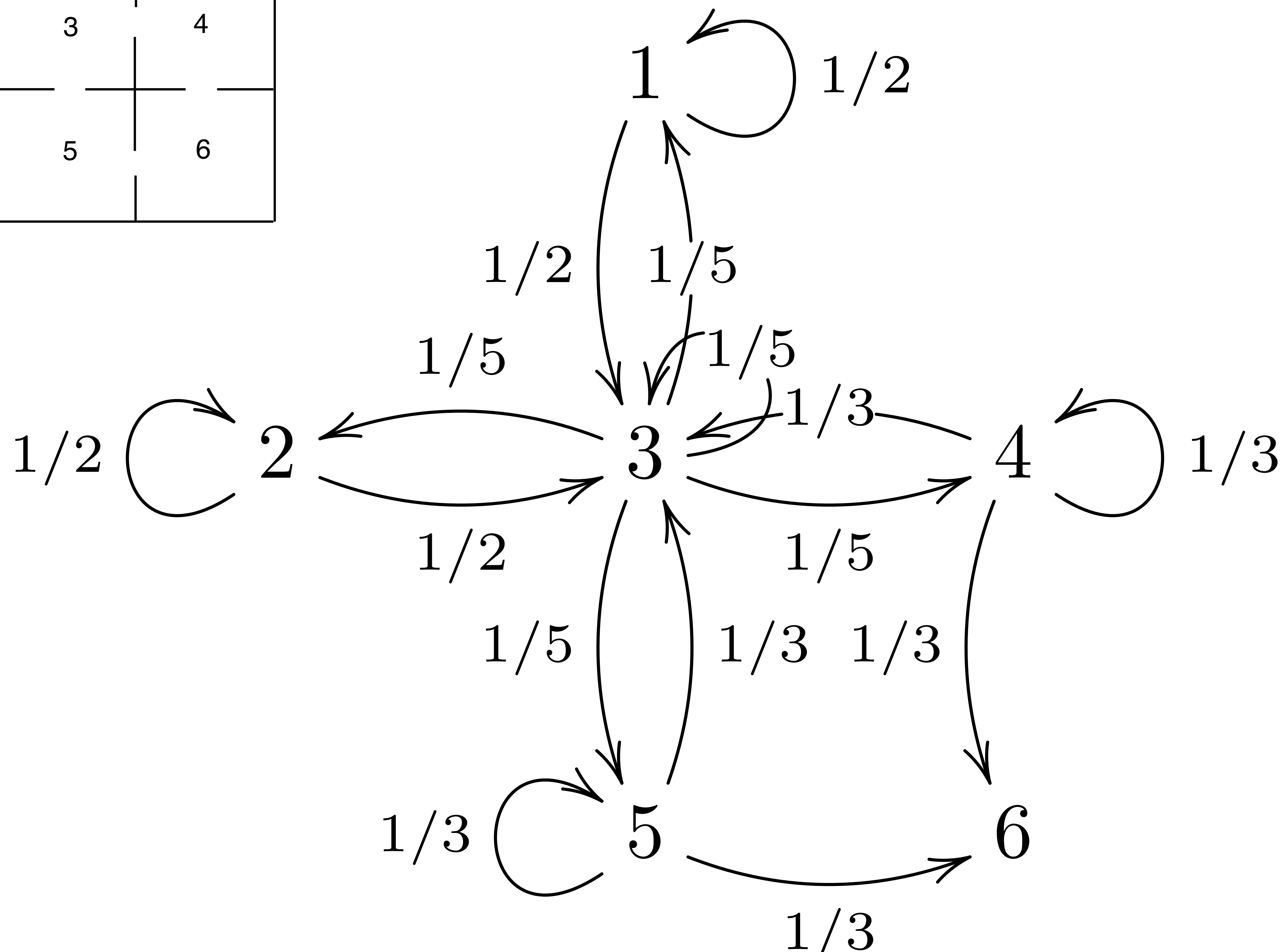
Ex. 2, mouse



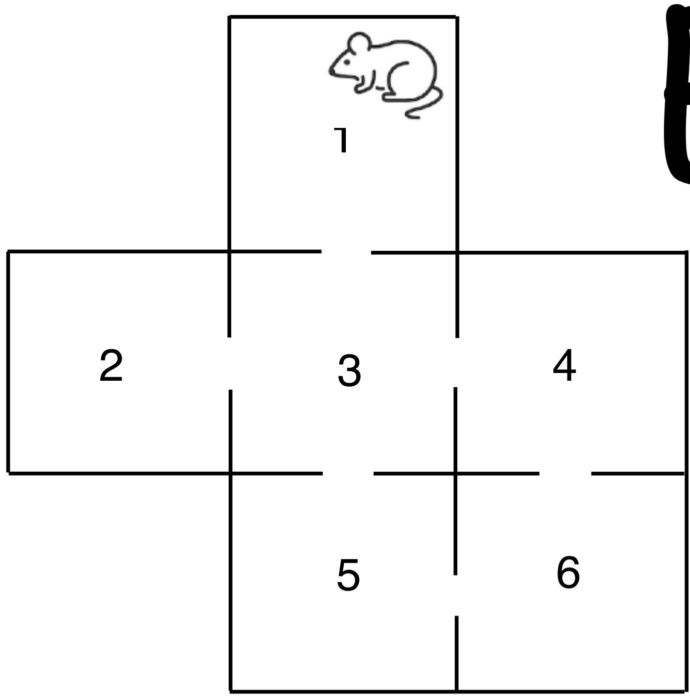


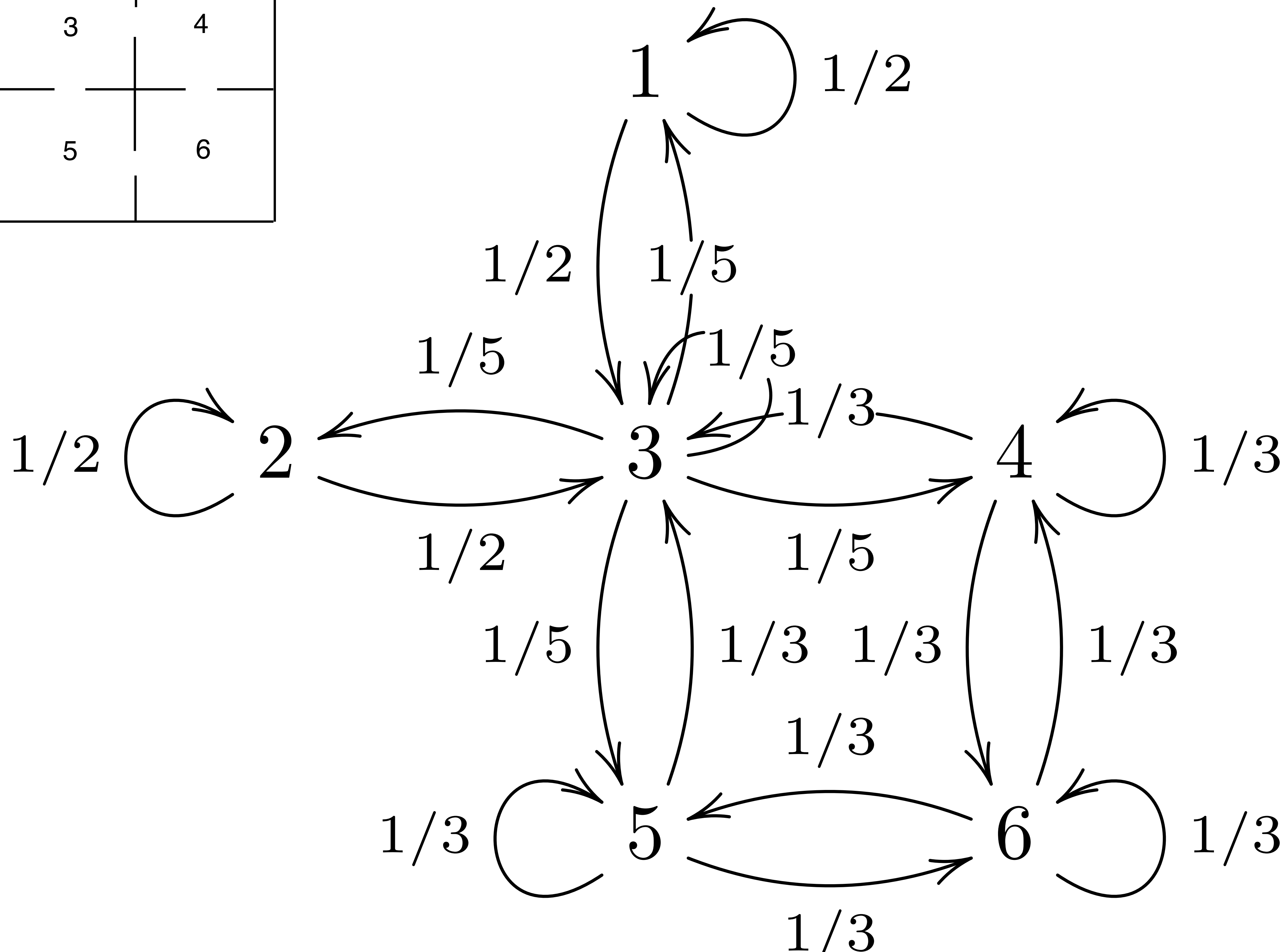
Ex. 2, mouse



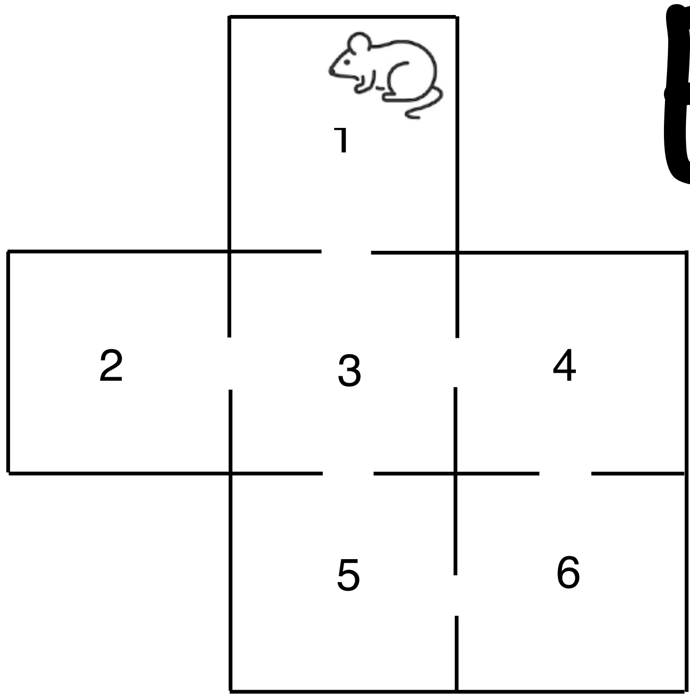


Ex. 2, mouse





Ex. 2, mouse



$$P = \begin{bmatrix} 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 & 0 & 0 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 & 0 \\ 0 & 0 & 1/3 & 1/3 & 0 & 1/3 \\ 0 & 0 & 1/3 & 0 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \end{bmatrix}$$

ergodic

Ex. 2, mouse

$$\begin{cases} \pi = \pi \cdot P \\ \sum_{i=1}^{N} \pi_i = 1 \end{cases} \quad \begin{cases} \pi_1 &=& \pi_1/2 + \pi_3/5 \\ \pi_2 &=& \pi_2/2 + \pi_3/5 \\ \pi_3 &=& \pi_1/2 + \pi_2/2 + \pi_3/5 + \pi_4/3 + \pi_5/3 \\ \pi_4 &=& \pi_3/5 + \pi_4/3 + \pi_6/3 \\ \pi_5 &=& \pi_3/5 + \pi_5/3 + \pi_6/3 \\ \pi_6 &=& \pi_4/3 + \pi_5/3 + \pi_6/3 \\ 1 &=& \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 \end{cases}$$

$$\begin{cases} \pi_1 &=& 2\pi_3/5 \\ \pi_2 &=& 2\pi_3/5 \\ 2\pi_3/5 &=& (\pi_4 + \pi_5)/3 \\ \pi_6 &=& 3\pi_3/5 \\ \pi_4 &=& 3\pi_3/5 \\ \pi_5 &=& 3\pi_3/5 \\ 1 &=& 18\pi_3/5 \end{cases}$$

$$\pi = \left[\frac{1}{9}, \frac{1}{9}, \frac{5}{18}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6} \right]$$

$$\pi = \left[\frac{2}{18}, \frac{2}{18}, \frac{5}{18}, \frac{3}{18}, \frac{3}{18}, \frac{3}{18} \right]$$

Ex. 2, mouse

$$P = \begin{bmatrix} 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 & 0 & 0 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 & 0 \\ 0 & 0 & 1/3 & 1/3 & 0 & 1/3 \\ 0 & 0 & 1/3 & 0 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$\pi^{(0)} = [\ 1\ ,\ 0\ ,\ 0\ ,\ 0\ ,\ 0\ ,\ 0\ ]$$

$$\pi^{(1)} = \pi^{(0)} \cdot P = \left[\ \frac{1}{2}\ ,\ 0\ ,\ \frac{1}{2}\ ,\ 0\ ,\ 0\ ,\ 0\ \right]$$

$$\pi^{(2)} = \pi^{(1)} \cdot P = \left[\ \frac{7}{20}\ ,\ \frac{1}{10}\ ,\ \frac{7}{20}\ ,\ \frac{1}{10}\ ,\ \frac{1}{10}\ ,\ 0\ \right]$$

$$\pi^{(3)} = \pi^{(2)} \cdot P = \left[\ \cdots\ ,\ \cdots\ ,\ \cdots\ ,\ \cdots\ ,\ \cdots\ ,\ \boxed{\frac{1}{15}}\ \right]$$

Alternatively: there are two paths of length 3 to room 6

$$P(1\ 3\ 4\ 6) = \frac{1}{2} \cdot \frac{1}{5} \cdot \frac{1}{3} = \frac{1}{30} \qquad P(1\ 3\ 5\ 6) = \frac{1}{2} \cdot \frac{1}{5} \cdot \frac{1}{3} = \frac{1}{30}$$

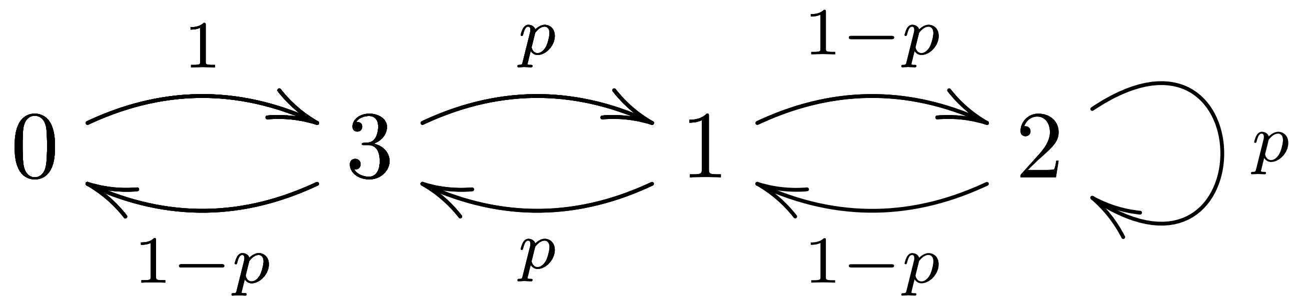
[**Ex. 3**] You have 3 umbrellas, some at home, some at the office. You keep moving between home and office.

- If it rains you take an umbrella with you from one place to another.
- If it does not rain you do not carry any umbrella.

It may happen that you must leave one place and it starts raining, but you do not have any umbrella with you, so you get wet.
Suppose you have been doing this for several years and that the probability of rain is (and has always been) p.
You are about to leave one place, what is the probability that you get wet? Use DTMCs to answer the question.
Hint: In the modelling of states, the fact that you are at home or in the office is irrelevant.

Ex. 3, umbrellas

state n: you are at one place with n umbrellas available



Ex. 3, umbrellas

state n: you are at one place with n umbrellas available

$$\begin{tikzcd}
0 \arrow[r, bend left, "1"] & 3 \arrow[l, bend left, "1-p"] \arrow[r, bend left, "p"] & 1 \arrow[l, bend left, "p"] \arrow[r, bend left, "1-p"] & 2 \arrow[l, bend left, "1-p"] \arrow[loop right, "p"]
\end{tikzcd}$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1-p & p \\ 0 & 1-p & p & 0 \\ 1-p & p & 0 & 0 \end{bmatrix}$$

ergodic

Ex. 3, umbrellas

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1-p & p \\ 0 & 1-p & p & 0 \\ 1-p & p & 0 & 0 \end{bmatrix}$$

we can compute the steady state distribution π

then π_0 is the probability to be at one place with no umbrella

and $p \cdot \pi_0$ is the probability to get wet

$$\begin{cases} \pi = \pi \cdot P \\ \sum_{i=1}^{N} \pi_i = 1 \end{cases}$$

$$\begin{cases} \pi_0 &=& (1-p) \cdot \pi_3 \\ \pi_1 &=& (1-p) \cdot \pi_2 + p \cdot \pi_3 \\ \pi_2 &=& (1-p) \cdot \pi_1 + p \cdot \pi_2 \\ \pi_3 &=& \pi_0 + p \cdot \pi_1 \\ 1 &=& \pi_0 + \pi_1 + \pi_2 + \pi_3 \end{cases}$$

$$\begin{cases} \pi_2 &=& \pi_1 \\ \pi_0 &=& (1-p) \cdot \pi_3 \\ \pi_1 &=& \pi_3 \\ 1 &=& (1-p) \cdot \pi_3 + \pi_3 + \pi_3 + \pi_3 \end{cases}$$

$$\Rightarrow \pi_3 = \frac{1}{4-p}$$

$$\pi_0 = \frac{1-p}{4-p}$$

$$\boxed{p \cdot \frac{1-p}{4-p}}$$

probability to get wet!

CTMC

[Ex. 4] Consider a CTMC with $N+1$ states, each representing the number of active instances of a service, from 0 to a maximum N. Let i denote the number of currently active instances. A new instance is spawned with rate

$$\lambda_i \stackrel{\text{def}}{=} (N-i) \times \lambda$$

for some fixed λ, i.e., the rate decreases as the number of instances already running increases,[1] while a running instance is terminated with rate

$$\mu_i \stackrel{\text{def}}{=} i \times \mu$$

for some fixed μ, i.e., the rate increases as there are more active instances to be terminated. Let $N = 3$, then:

1. Model the system as a CTMC.

2. Use the infinitesimal generator matrix to find the steady state probability distribution.

[1] Imagine the number of clients is fixed. When i instances of the service are already active to serve i clients, then the number of clients that can require a new instance of the service is decreased by i.

Ex. 4, CTMC

$$\lambda_0 = 3\lambda \qquad \mu_0 = 0$$

$$\lambda_1 = 2\lambda \qquad \mu_1 = \mu$$

$$\lambda_2 = \lambda \qquad \mu_2 = 2\mu$$

$$\lambda_3 = 0 \qquad \mu_3 = 3\mu$$

$$\begin{tikzcd}
0 \arrow[r, bend left, "3\lambda"] & 1 \arrow[l, bend left, "\mu"] \arrow[r, bend left, "2\lambda"] & 2 \arrow[l, bend left, "2\mu"] \arrow[r, bend left, "\lambda"] & 3 \arrow[l, bend left, "3\mu"]
\end{tikzcd}$$

$$Q = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}$$

Ex. 4, CTMC

$$\lambda_0 = 3\lambda \qquad \mu_0 = 0$$

$$\lambda_1 = 2\lambda \qquad \mu_1 = \mu$$

$$\lambda_2 = \lambda \qquad \mu_2 = 2\mu$$

$$\lambda_3 = 0 \qquad \mu_3 = 3\mu$$

$$\begin{tikzcd}
0 \arrow[r, bend left, "3\lambda"] & 1 \arrow[l, bend left, "\mu"] \arrow[r, bend left, "2\lambda"] & 2 \arrow[l, bend left, "2\mu"] \arrow[r, bend left, "\lambda"] & 3 \arrow[l, bend left, "\mu"]
\end{tikzcd}$$

$$Q = \begin{bmatrix} -3\lambda & 3\lambda & 0 & 0 \\ \mu & -\mu - 2\lambda & 2\lambda & 0 \\ 0 & 2\mu & -2\mu - \lambda & \lambda \\ 0 & 0 & 3\mu & -3\mu \end{bmatrix} \qquad \begin{cases} p \cdot Q = 0 \\ \sum_{i=1}^{N} p_i = 1 \end{cases}$$

Ex. 4, CTMC

$$Q = \begin{bmatrix} -3\lambda & 3\lambda & 0 & 0 \\ \mu & -\mu-2\lambda & 2\lambda & 0 \\ 0 & 2\mu & -2\mu-\lambda & \lambda \\ 0 & 0 & 3\mu & -3\mu \end{bmatrix}$$

$$\begin{cases} p \cdot Q = 0 \\ \displaystyle\sum_{i=1}^{N} p_i = 1 \end{cases}$$

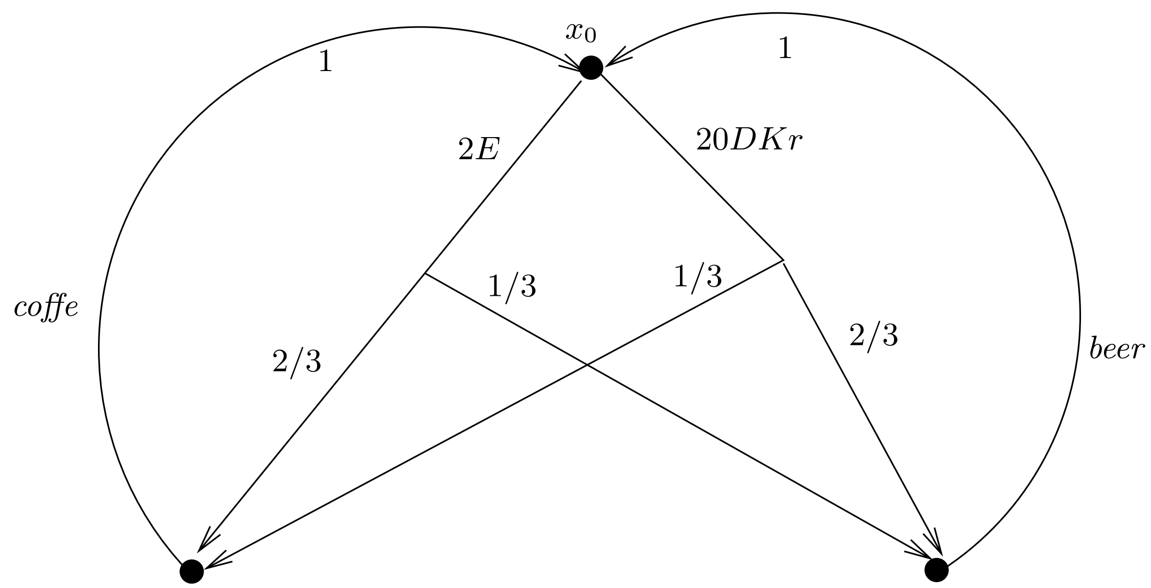
$$\begin{cases} 0 &= -3\lambda p_0 + \mu p_1 \\ 0 &= 3\lambda p_0 - \mu p_1 - 2\lambda p_1 + 2\mu p_2 \\ 0 &= 2\lambda p_1 - 2\mu p_2 - \lambda p_2 + 3\mu p_3 \\ 0 &= \lambda p_2 - 3\mu p_3 \\ 1 &= p_0 + p_1 + p_2 + p_3 \end{cases}$$

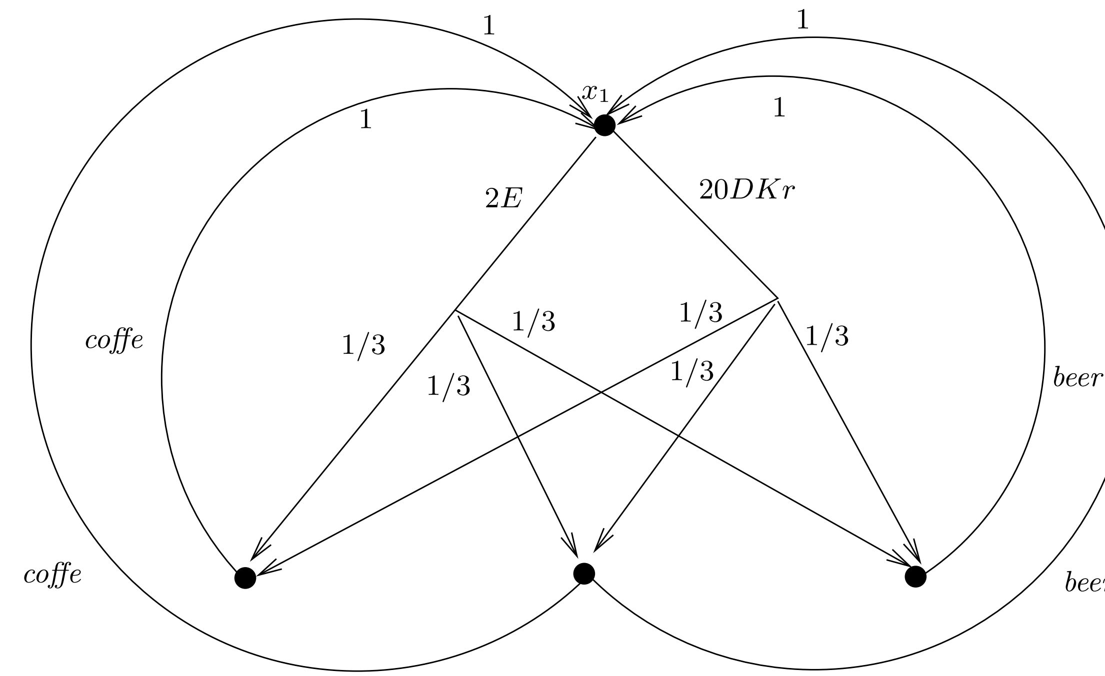
$$\begin{cases} p_1 &= \frac{3\lambda}{\mu} p_0 \\ p_2 &= \frac{3\lambda^2}{\mu^2} p_0 \\ p_3 &= \frac{\lambda^3}{\mu^3} p_0 \\ 1 &= p_0 + \frac{3\lambda}{\mu} p_0 + \frac{3\lambda^2}{\mu^2} p_0 + \frac{\lambda^3}{\mu^3} p_0 \qquad \Rightarrow p_0 = \frac{\mu^3}{(\lambda+\mu)^3} \end{cases}$$

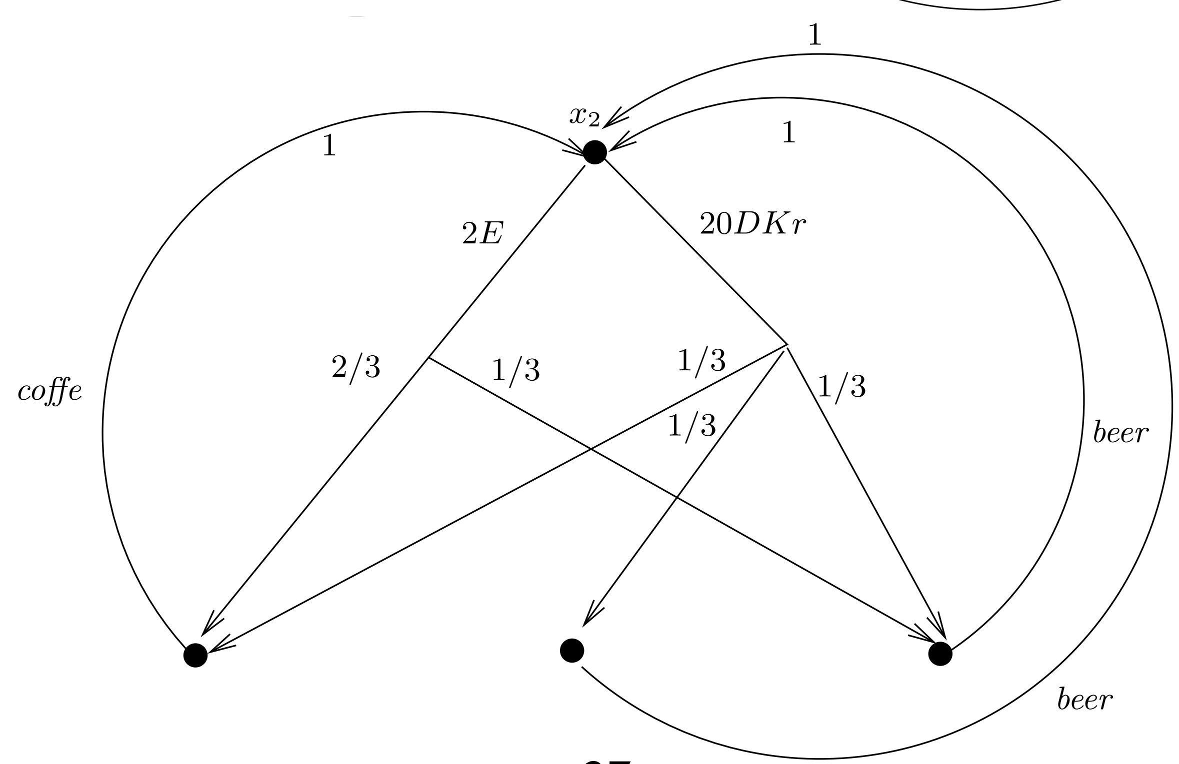
$$\boxed{p = \left[\frac{\mu^3}{(\lambda+\mu)^3}, \frac{3\lambda\mu^2}{(\lambda+\mu)^3}, \frac{3\lambda^2\mu}{(\lambda+\mu)^3}, \frac{\lambda^3}{(\lambda+\mu)^3} \right]}$$

PTS

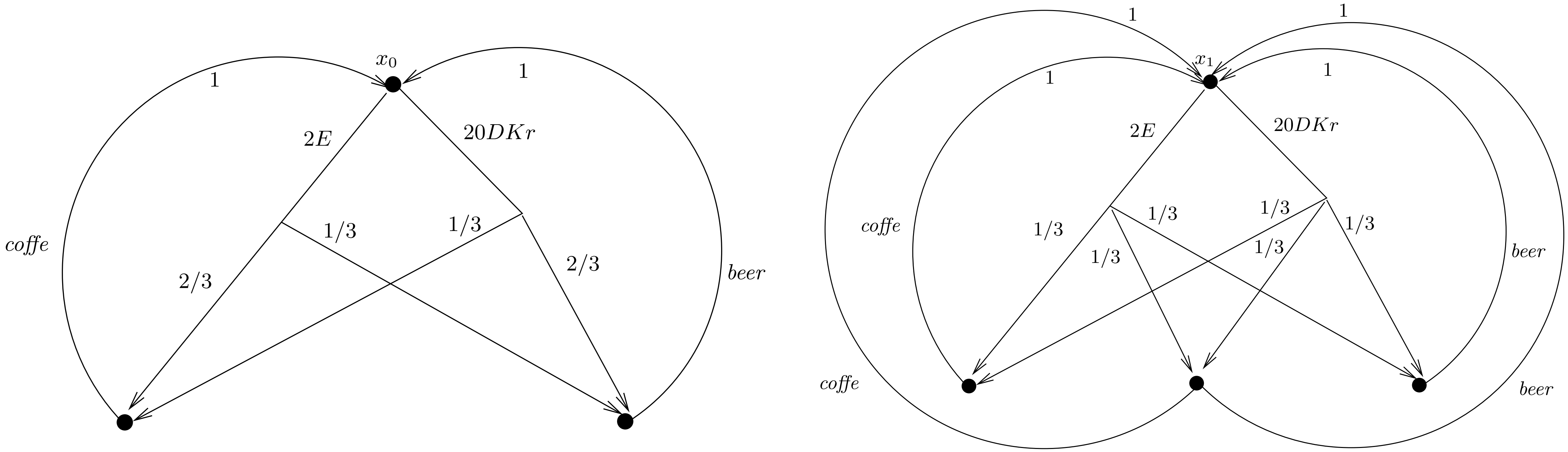
[**Ex. 5**] Consider the following reactive PTSs. For every pair of systems, check whether their initial states are bisimilar. If they are, describe the bisimulation, if they are not, find a formula of the Larsen-Skou logic that distinguishes them.







Ex. 5, reactive PTS

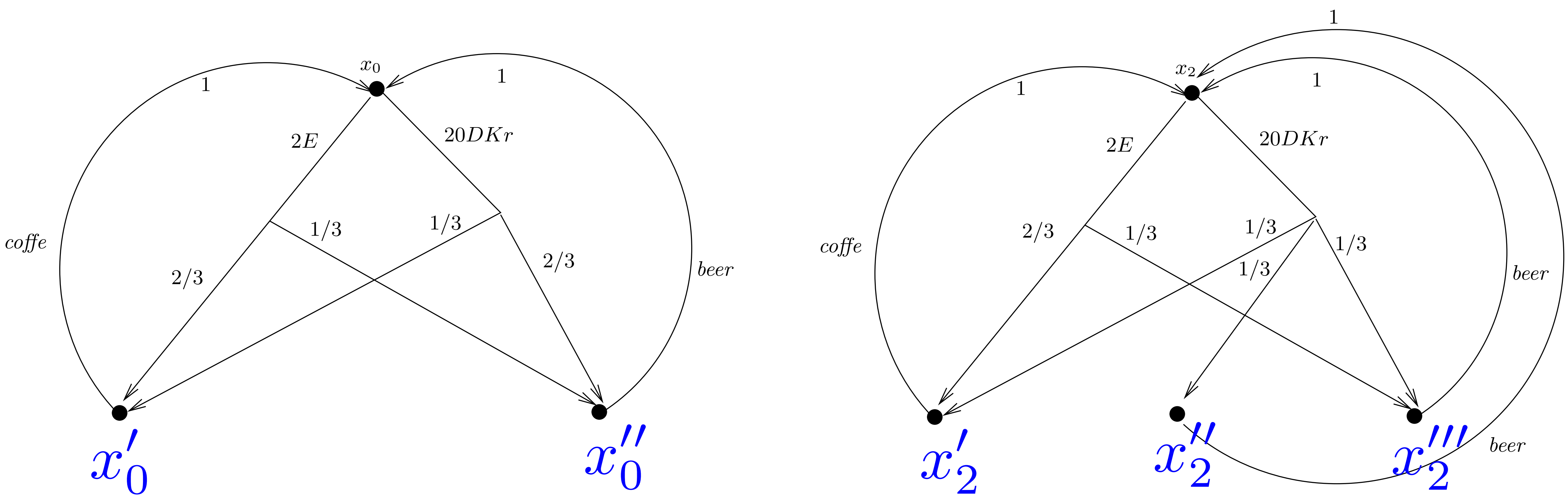


$$x_0 \not\simeq_R x_1$$

$$\varphi \triangleq \langle 2E \rangle_{\frac{1}{3}} (\langle \mathit{coffee} \rangle_1 \mathbf{tt} \wedge \langle \mathit{beer} \rangle_1 \mathbf{tt})$$

$x_0 \not\models \varphi$ $\qquad\qquad$ $x_1 \models \varphi$

Ex. 5, reactive PTS



$$x_0 \simeq_R x_2$$

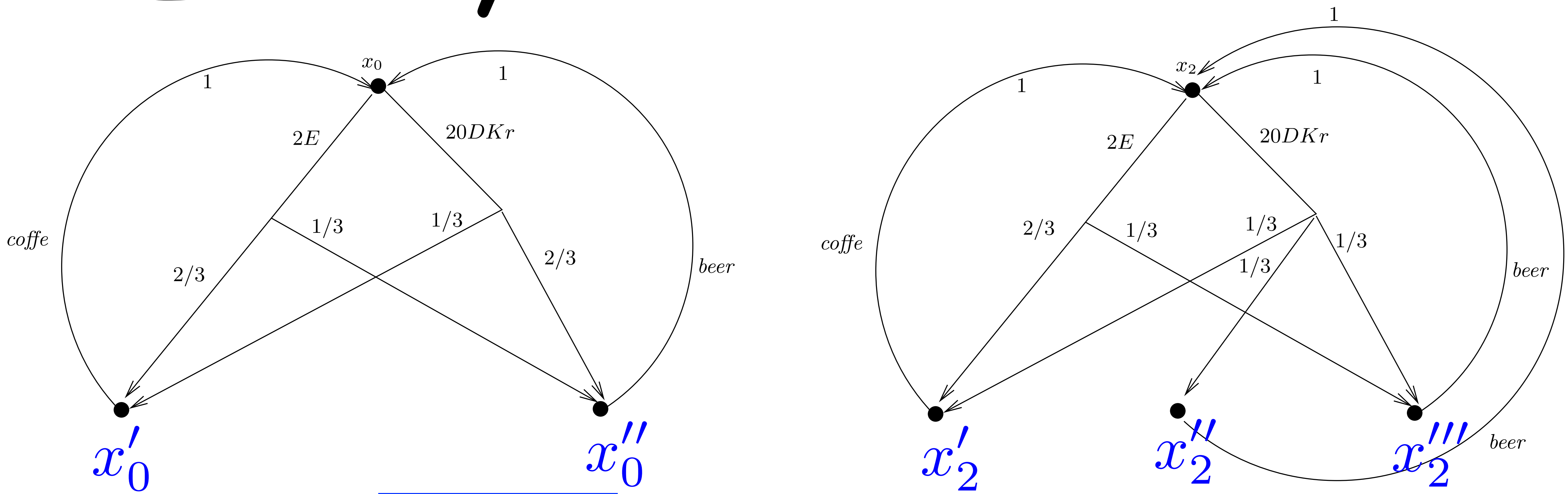
$$\mathbf{R} = \{\ \{x_0, x_2\}\ ,\ \{x_0', x_2'\}\ ,\ \{x_0'', x_2'', x_2'''\}\ \}$$

$$I_1 \qquad\qquad I_2 \qquad\qquad I_3$$

is a reactive bisimulation

let's check just the cases where $\gamma_R(s, \ell, I) \neq 0$

Ex. 5, reactive PTS



$$\mathbf{R} = \{ \boxed{\underset{I_1}{\{x_0, x_2\}}}, \underset{I_2}{\{x'_0, x'_2\}}, \underset{I_3}{\{x''_0, x''_2, x'''_2\}} \}$$

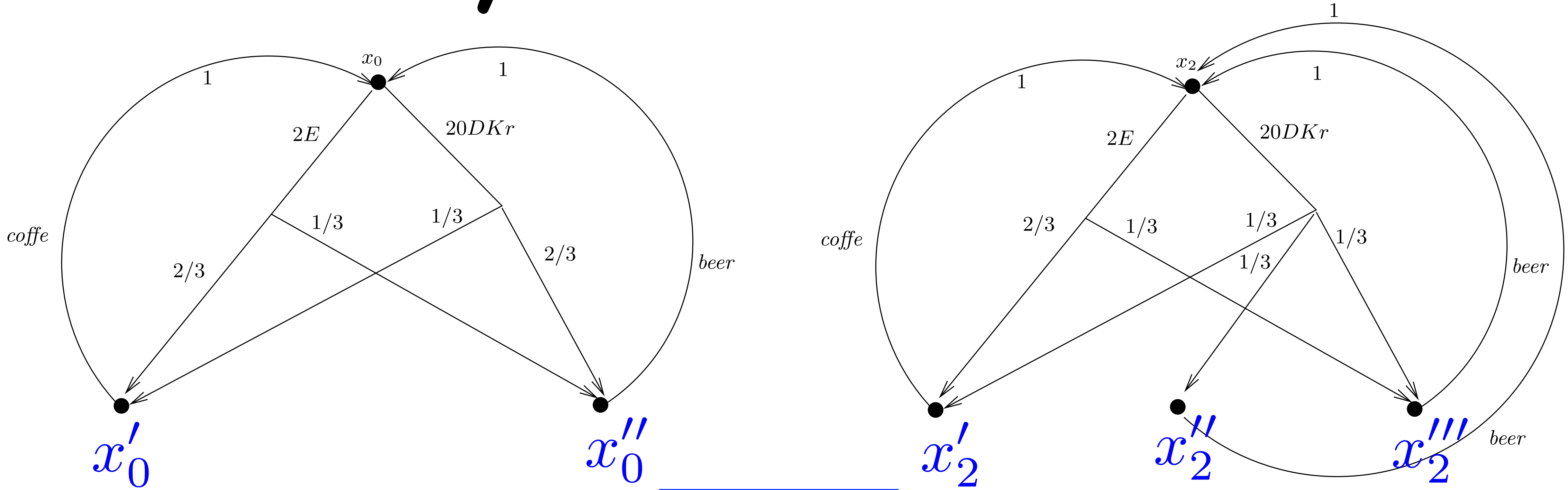
$$\gamma_R(x_0, 2E, I_2) = 2/3 \qquad \gamma_R(x_2, 2E, I_2) = 2/3$$

$$\gamma_R(x_0, 2E, I_3) = 1/3 \qquad \gamma_R(x_2, 2E, I_3) = 1/3$$

$$\gamma_R(x_0, 20Dkr, I_2) = 1/3 \qquad \gamma_R(x_2, 20Dkr, I_2) = 1/3$$

$$\gamma_R(x_0, 20Dkr, I_3) = 2/3 \qquad \gamma_R(x_2, 20Dkr, I_3) = 2/3$$

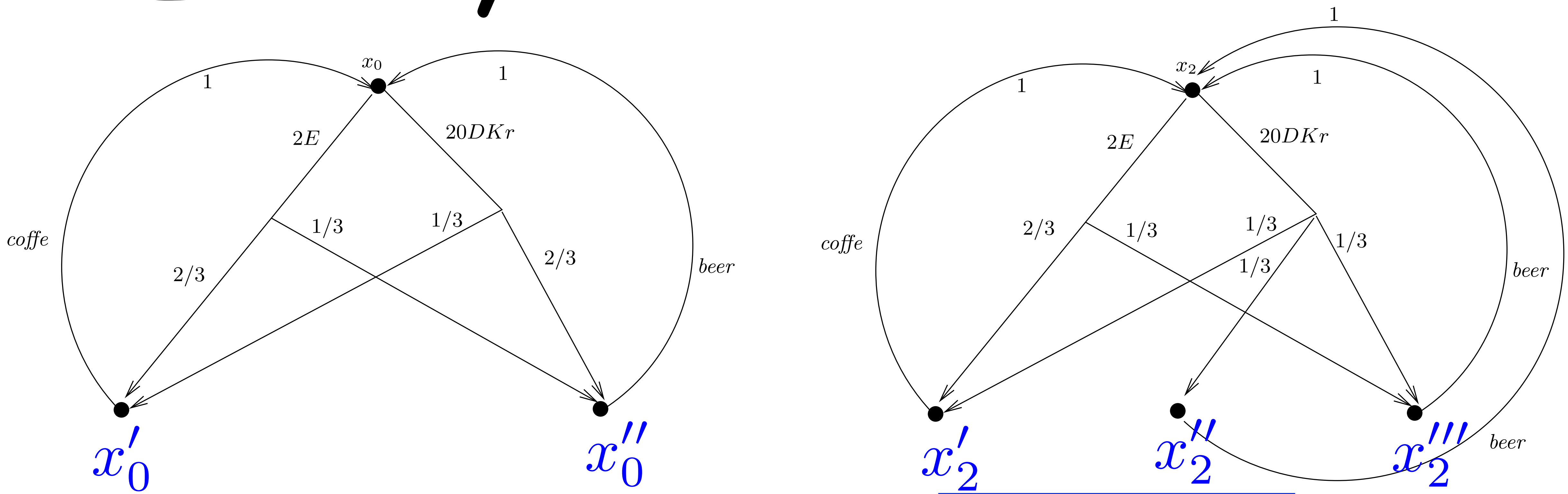
Ex. 5, reactive PTS



$$\mathbf{R} = \{\ \underset{I_1}{\{x_0, x_2\}}\ ,\ \boxed{\underset{I_2}{\{x_0', x_2'\}}}\ ,\ \underset{I_3}{\{x_0'', x_2'', x_2'''\}}\ \}$$

$$\gamma_R(x_0', \mathit{coffee}, I_1) = 1 \qquad\qquad \gamma_R(x_2', \mathit{coffee}, I_1) = 1$$

Ex. 5, reactive PTS



$$\mathbf{R} = \{\ \underset{I_1}{\{x_0, x_2\}}\ ,\ \underset{I_2}{\{x_0', x_2'\}}\ ,\ \boxed{\underset{I_3}{\{x_0'', x_2'', x_2'''\}}}\ \}$$

$$\gamma_R(x_0'', beer, I_1) = 1 \quad \gamma_R(x_2'', beer, I_1) = 1 \quad \gamma_R(x_2''', beer, I_1) = 1$$

PEPA

[**Ex. 6**] Use PEPA to model the service system of Ex. 4 (for $N = 3$), using action *new* for spawning a new instance and action *end* to terminate it. Design a second PEPA process to model a system composed by 3 independent service components, each with $N = 1$. Are the two systems CTMC bisimilar?

Ex. 6, PEPA

Ex 4

$$\begin{tikzcd} 0 \arrow[r, bend left, "3\lambda"] & 1 \arrow[l, bend left, "\mu"] \arrow[r, bend left, "2\lambda"] & 2 \arrow[l, bend left, "2\mu"] \arrow[r, bend left, "\lambda"] & 3 \arrow[l, bend left, "\mu"] \end{tikzcd}$$

$$\begin{aligned} \mathsf{S}_0^3 &\triangleq (new, 3\lambda).\mathsf{S}_1^3 \\ \mathsf{S}_1^3 &\triangleq (new, 2\lambda).\mathsf{S}_2^3 + (end, \mu).\mathsf{S}_0^3 \\ \mathsf{S}_2^3 &\triangleq (new, \lambda).\mathsf{S}_3^3 + (end, 2\mu).\mathsf{S}_1^3 \\ \mathsf{S}_3^3 &\triangleq (end, \mu).\mathsf{S}_2^3 \end{aligned}$$

$$\begin{aligned} \mathsf{S}_0^1 &\triangleq (new, \lambda).\mathsf{S}_1^1 \\ \mathsf{S}_1^1 &\triangleq (end, \mu).\mathsf{S}_0^1 \end{aligned}$$

$$\mathsf{S}_0^3 \overset{?}{\simeq}_C \mathsf{S}_0^1 \parallel \mathsf{S}_0^1 \parallel \mathsf{S}_0^1$$

Ex. 6, PEPA

$$\begin{tikzcd}
\mathsf{S}_0^3 \arrow[d, shift left, "{(new, 3\lambda)}"] \\
\mathsf{S}_1^3 \arrow[u, shift left, "{(end, \mu)}"] \arrow[d, shift left, "{(new, 2\lambda)}"] \\
\mathsf{S}_2^3 \arrow[u, shift left, "{(end, 2\mu)}"] \arrow[d, shift left, "{(new, \lambda)}"] \\
\mathsf{S}_3^3 \arrow[u, shift left, "{(end, 3\mu)}"]
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathsf{S}_0^1 \arrow[d, shift left, "{(new, \lambda)}"] \\
\mathsf{S}_1^1 \arrow[u, shift left, "{(end, \mu)}"]
\end{tikzcd}$$

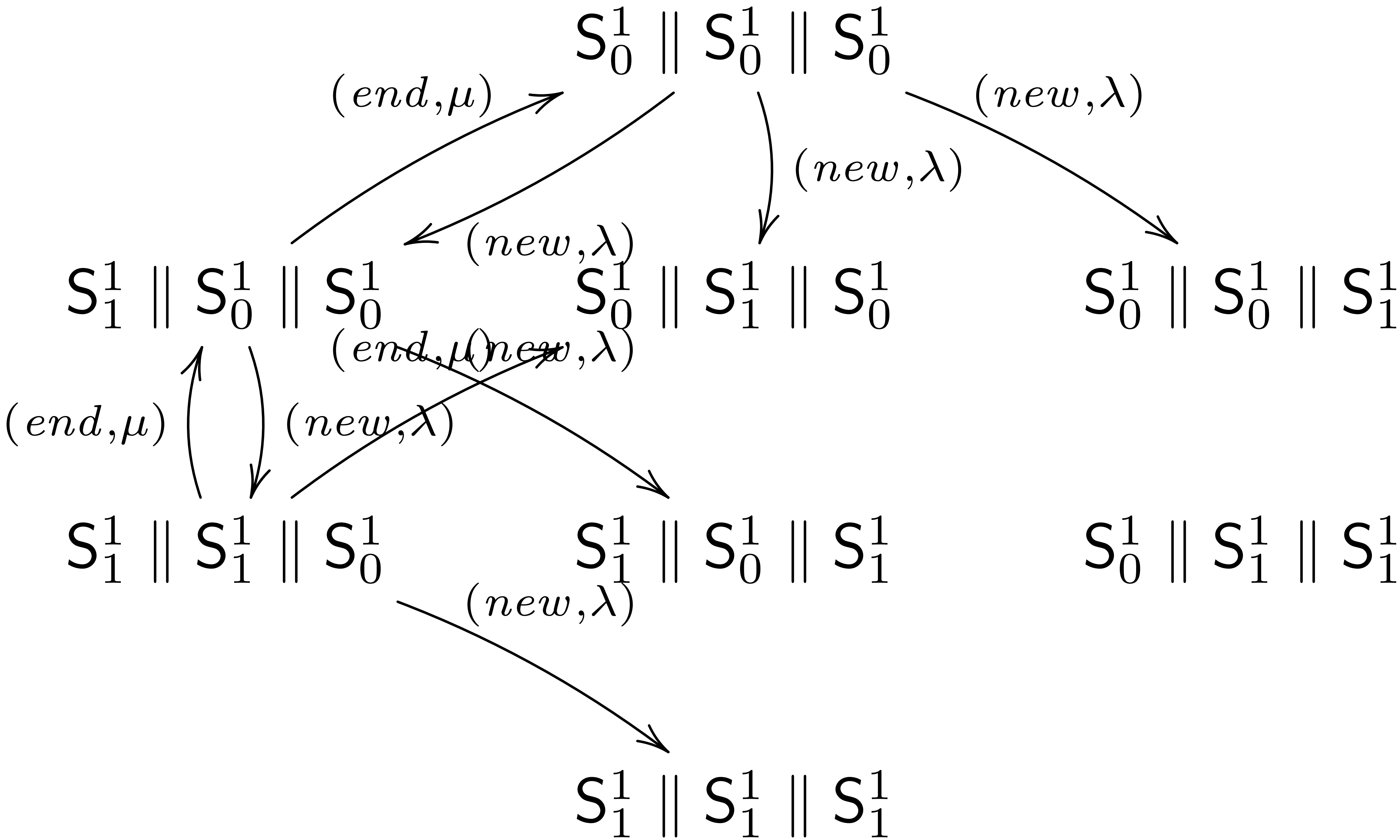
Ex. 6, PEPA

$$\begin{tikzcd}[column sep=small]
 & \mathsf{S}_0^1 \parallel \mathsf{S}_0^1 \parallel \mathsf{S}_0^1 \arrow[dl, "{(new,\lambda)}"] \arrow[d, "{(new,\lambda)}"] \arrow[dr, "{(new,\lambda)}"] & \\
\mathsf{S}_1^1 \parallel \mathsf{S}_0^1 \parallel \mathsf{S}_0^1 & \mathsf{S}_0^1 \parallel \mathsf{S}_1^1 \parallel \mathsf{S}_0^1 & \mathsf{S}_0^1 \parallel \mathsf{S}_0^1 \parallel \mathsf{S}_1^1
\end{tikzcd}$$

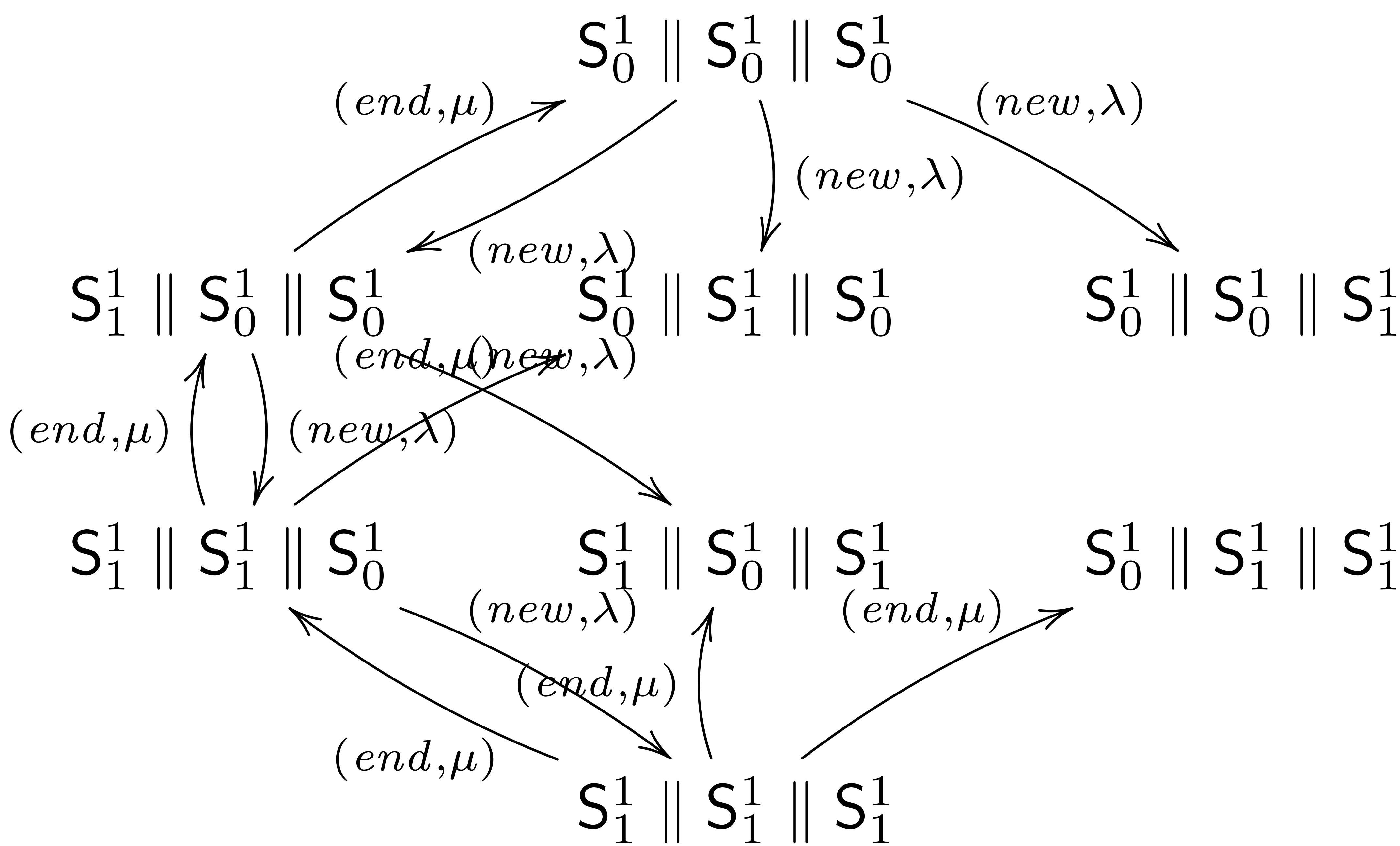
Ex. 6, PEPA

$$\begin{tikzcd}
 & \mathsf{S}_0^1 \parallel \mathsf{S}_0^1 \parallel \mathsf{S}_0^1 \arrow[dl, bend right, "(new, \lambda)"'] \arrow[d, "(new, \lambda)"] \arrow[dr, "(new, \lambda)"] & \\
\mathsf{S}_1^1 \parallel \mathsf{S}_0^1 \parallel \mathsf{S}_0^1 \arrow[ur, bend left, "(end, \mu)"] \arrow[d, "(new, \lambda)"] \arrow[dr, "(new, \lambda)"] & \mathsf{S}_0^1 \parallel \mathsf{S}_1^1 \parallel \mathsf{S}_0^1 & \mathsf{S}_0^1 \parallel \mathsf{S}_0^1 \parallel \mathsf{S}_1^1 \\
\mathsf{S}_1^1 \parallel \mathsf{S}_1^1 \parallel \mathsf{S}_0^1 & \mathsf{S}_1^1 \parallel \mathsf{S}_0^1 \parallel \mathsf{S}_1^1 & \mathsf{S}_0^1 \parallel \mathsf{S}_1^1 \parallel \mathsf{S}_1^1
\end{tikzcd}$$

Ex. 6, PEPA

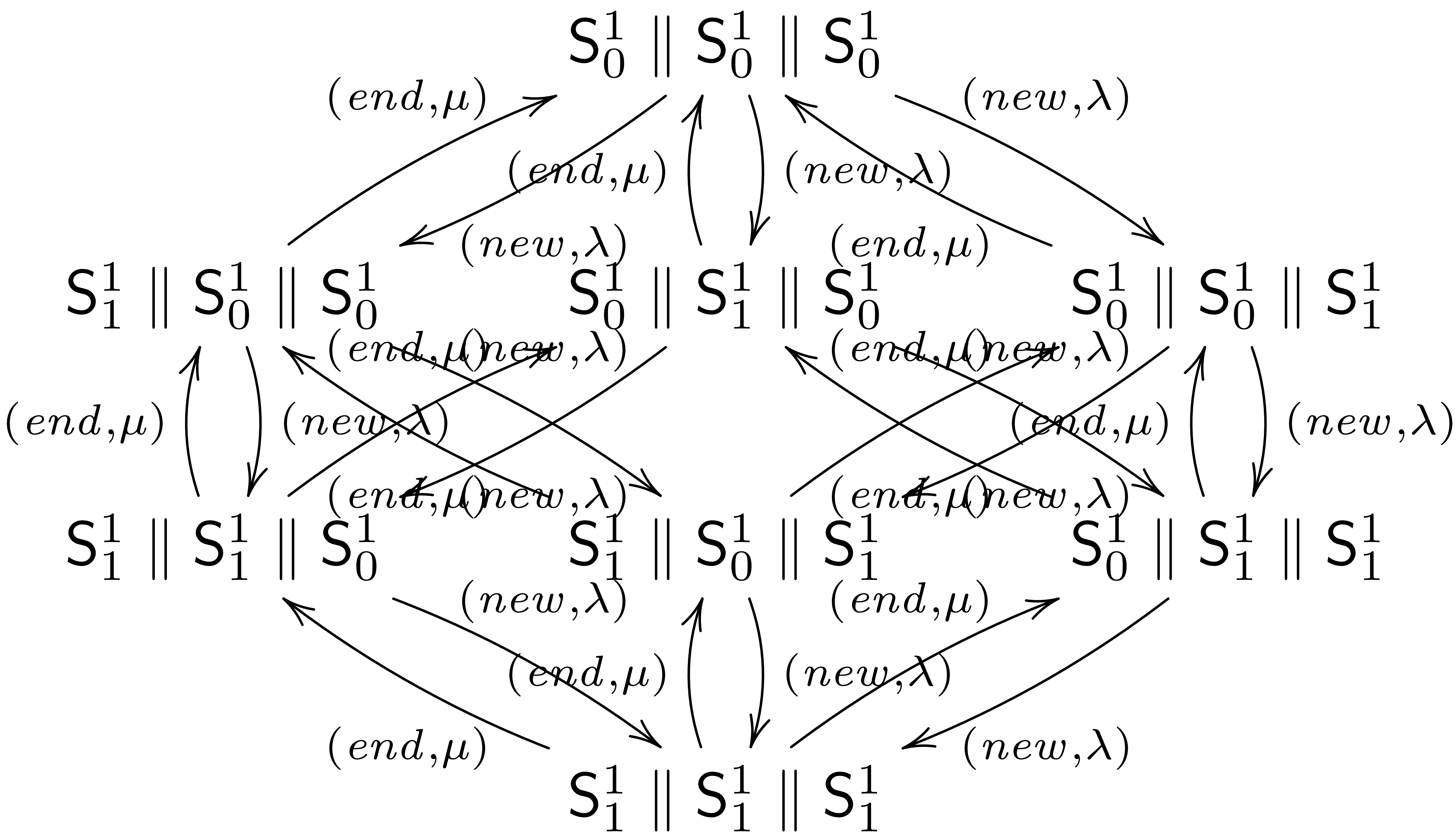

$S^1_0 \parallel S^1_0 \parallel S^1_0$
(end, μ)
(new, λ)
(new, λ)
(new, λ)
$S^1_1 \parallel S^1_0 \parallel S^1_0$
$S^1_0 \parallel S^1_1 \parallel S^1_0$
$S^1_0 \parallel S^1_0 \parallel S^1_1$
(end, μ)
(new, λ)
(end, μ)
(new, λ)
$S^1_1 \parallel S^1_1 \parallel S^1_0$
$S^1_1 \parallel S^1_0 \parallel S^1_1$
$S^1_0 \parallel S^1_1 \parallel S^1_1$
(new, λ)
$S^1_1 \parallel S^1_1 \parallel S^1_1$

Ex. 6, PEPA



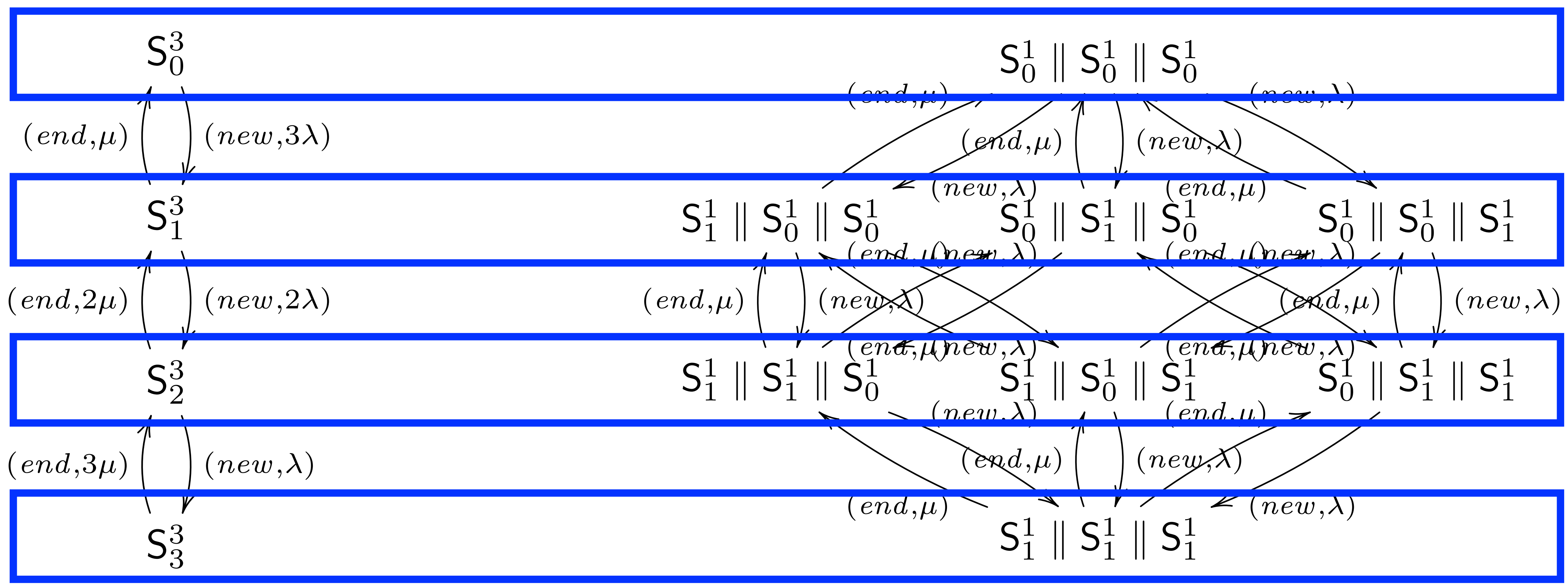
$S^1_0 \parallel S^1_0 \parallel S^1_0$
(end, μ)
(new, λ)
(new, λ)
(new, λ)
$S^1_1 \parallel S^1_0 \parallel S^1_0$
$S^1_0 \parallel S^1_1 \parallel S^1_0$
$S^1_0 \parallel S^1_0 \parallel S^1_1$
(end, μ)
(new, λ)
(end, μ)
(new, λ)
$S^1_1 \parallel S^1_1 \parallel S^1_0$
$S^1_1 \parallel S^1_0 \parallel S^1_1$
$S^1_0 \parallel S^1_1 \parallel S^1_1$
(new, λ)
(end, μ)
(end, μ)
(end, μ)
$S^1_1 \parallel S^1_1 \parallel S^1_1$

Ex. 6, PEPA



$S_0^1 \parallel S_0^1 \parallel S_0^1$
$S_1^1 \parallel S_0^1 \parallel S_0^1$
$S_0^1 \parallel S_1^1 \parallel S_0^1$
$S_0^1 \parallel S_0^1 \parallel S_1^1$
$S_1^1 \parallel S_1^1 \parallel S_0^1$
$S_1^1 \parallel S_0^1 \parallel S_1^1$
$S_0^1 \parallel S_1^1 \parallel S_1^1$
$S_1^1 \parallel S_1^1 \parallel S_1^1$
(end, μ)
(new, λ)

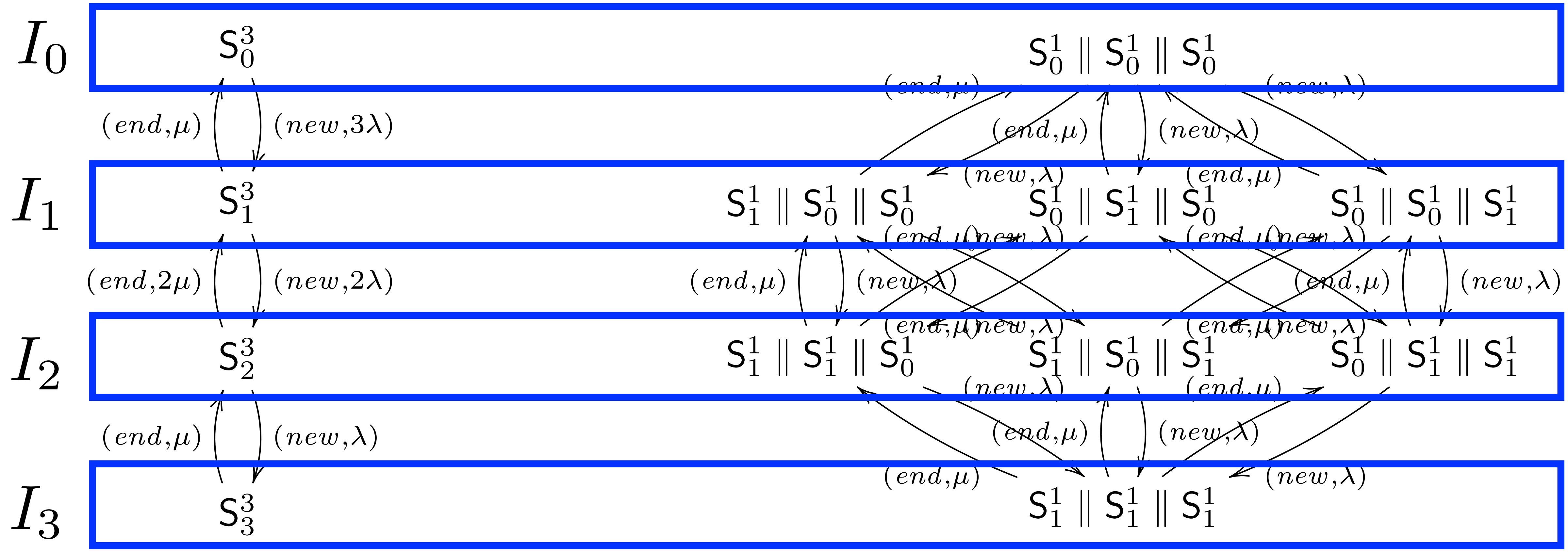
Ex. 6, PEPA



$$
\begin{aligned}
\mathbf{R} = \{\ & \{S_0^3\,,\ S_0^1 \parallel S_0^1 \parallel S_0^1\}\,,\\
& \{S_1^3\,,\ S_1^1 \parallel S_0^1 \parallel S_0^1\,,\ S_0^1 \parallel S_1^1 \parallel S_0^1\,,\ S_0^1 \parallel S_0^1 \parallel S_1^1\}\,,\\
& \{S_2^3\,,\ S_1^1 \parallel S_1^1 \parallel S_0^1\,,\ S_1^1 \parallel S_0^1 \parallel S_1^1\,,\ S_0^1 \parallel S_1^1 \parallel S_1^1\}\,,\\
& \{S_3^3\,,\ S_1^1 \parallel S_1^1 \parallel S_1^1\}\ \}
\end{aligned}
$$

is a CTMC bisimulation

Ex. 6, PEPA



$$\gamma_C(\mathsf{S}_1^3, I_0) = \mu \qquad \gamma_C(\mathsf{S}_1^1 \parallel \mathsf{S}_0^1 \parallel \mathsf{S}_0^1, I_0) = \mu$$

$$\gamma_C(\mathsf{S}_1^3, I_2) = 2\lambda \qquad \gamma_C(\mathsf{S}_1^1 \parallel \mathsf{S}_0^1 \parallel \mathsf{S}_0^1, I_2) = 2\lambda$$

etc.